ANALYSIS OF TIME SERIES AND FORECASTING ON THE BASIS OF STATIONARY FUZZY-MARKOV PROCESSES

IDŐSOROK ANALÍZISE ÉS ELŐREJELZÉSEK KÉSZÍTÉSE STACIONÁRIUS FUZZY-MARKOV FOLYAMATOK ALAPJÁN

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Abstract

Time series data can be found in various fields of science and in real life also. Fundamental question is that what kind of mathematical model could be applied for the description and for the analysis of observations, and how could predictions be given for future. Basic expectation against the model, that for observations an acceptable level of accuracy is a requirement, which is determined by the user or the customer. If the level of accuracy is acceptable forecasts can be made for the future. Considering this topic, several mathematical models and tools can be found in the literature. In this paper the unification of two efficient theories, the theory of stochastic processes and fuzzy logic will be presented, that is the so-called fuzzy-Markov process. The application of this tool will be presented for approximation and for forecasting in case of a real life time series.

Keywords

time series, forecasting, Markov-process, fuzzy logic

Absztrakt

Az élet, az alkalmazott és egzakt tudományok számos területén találkozunk idősorokkal. E témában az az alapvető kérdés, hogy hogyan lehetséges a megfigyelések során kapott adatsorra egy olyan matematikai modellt felállítani, amely a felhasználó által előírt hibahatáron belül szolgáltatja a megfigyelt adatokat is és amely modell alapján előrejelzéseket lehet készíteni az idősor további adataira a megfigyeléseket sorozatát követő néhány időpontjára vonatkozólag. A témakörben számos módszer fellelhető a szakirodalomban. Ebben a cikkben két nagy horderejű matematika elméletnek, a Markov-láncok elméletének és a fuzzy logikának az egyesítésével kapott modellel, egy speciális fuzzy-Markov folyamattal fogjuk modellezni az idősort és ennek alapján adunk majd előrejelzést a jövőre nézve.

Kulcsszavak

idősor, előrejelzés, Markov-folyamat, fuzzy logika

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INTRODUCTION

In security science there are lots of problems, that can be modeled by times series, as a mathematical model. For example in cyber security the annual, monthly, weekly or daily number of attacks against one computer network. In food security, for example the annual amount of cereal crop in any country, or the annual amount of exported crop from Ukraine, from China, etc. In traffic security, the yearly, monthly, etc. number of car accidents, train accidents, etc. in any country. A workplace security (or health security) problem for example the yearly number of workplace accidents in any county. And the long list could be continued. These data series are called time series in mathematics. Therefore in previously mentioned cases self evident choice would be the so-called "time series analysis" and forecasting, as an appropriate mathematical model. A basic expectation against this kind of mathematical model, on the one hand that observed data must be approximated accurately, and if this condition is fulfilled, providing predictions for some years in the future.

Several mathematical methods have been developed for this purpose [1], [2], [3], in which mostly the tools of calculus and probability theory are used. In this paper a different possible approach will be presented, on the basis of the unification of two great mathematical theories, the theory of stochastic processes, more precisely theory of Markov-chains, and the fuzzy logic [4], [5], [6], [7].

The structure of this article is as follows: First of all a real life time series will be demonstrated, which is the annual number of workplace accidents in Hungary since 2000. Then a short extract of the applied mathematical tools, Markov-chains and fuzzy logic will be presented, and applied for the observed data. The following topic will be the unification process of these tools for the time series and the prediction procedure will be explained. Finally analysis of the proposed tools and conclusions can be read.

PRESENTATION OF THE TIME SERIES

Before illustrating the theory and presenting the applied mathematical tool, a real-life time series is demonstrated. This time series presents the annual number of workplace accidents in Hungary from 2020. In table 1. observed data are summarized, and yearly change of these data are also highlighted.

year	Annual number of workplace accidents: $Y(n)$	Yearly change: $DY(n)$
2000	28 220	–
2001	26 369	-1 851
2002	26 072	-297
2003	26 392	320
2004	24 355	-2 037
2005	24 346	-9

year	Annual number of workplace accidents: $Y(n)$	Yearly change: $\mathrm{D}Y(n)$
2006	23 038	-1 308
2007	21 154	-1 884
2008	22 458	1 304
2009	18 693	-3 765
2010	20 123	1 430
2011	17 448	-2 675
2012	17 164	-284
2013	17 361	197
2014	19 787	2 426
2015	21 165	1 378
2016	23 027	1 862
2017	23 387	360
2018	23 738	351
2019	24 055	317
2020	20 366	-3 689
2021	21 591	1 225
2022	21 273	-318

Table 1.Yearly number of workplace accidents in Hungary, since 2000.
Source: https://www.ksh.hu/stadat_files/ege/hu/ege0042.html

In table 1. there are time series data ($Y(n)$) between 2000 and 2022, therefore observations exist for the last 23 years. The mathematical problem, on the one hand, is modelling the existing data, on the other hand forecasting, in other words, making reliable predictions for the future, more precisely for example for years 2023., 2024. and 2025. As it can be seen, in table 1. not only observed time series data are presented, but also the yearly change of these data. The accurate definition of the yearly change of $Y(n)$ is as follows: $\mathrm{D}Y(n) = Y(n)–Y(n–1)$ if $n \geq 2021$. Therefore, this change can be defined first for the second observed year. This difference plays fundamental role in the theory, it will be presented in the following sections, that this difference provides be the basis of the application of the fuzzy-Markov model [4], [5]. The examined time series is depicted in figure 1.

The first step in several commonly applied time series examination methods [1], [3] would be the classification of the time series. The question would be whether a typical increasing or decreasing trend can be identified or some kind of seasonality – in other words a periodic behavior – can be observed in the time series. In such case the procedure would be the decomposition of the time series for trend and seasonal components. The examined time series can't be characterized by neither of these properties. It can be said, that if such behavior can be observed, the modelling and the predicting procedure for the data is simpler. This kind of times series, without trend and periodicity intentionally has been chosen, because in this paper the author would like to present the efficiency of the fuzzy-Markov process. This is precisely the advantage of the fuzzy-Markov process. Instead of modelling trend and seasonal components the applied mathematical tool uses DY(n) differences for characterizing the time series. First of all the basic concept of the Markov-process will be presented in the following section.

Figure 1. Yearly number of workplace accidents in Hungary between 2000 and 2022.
Source: edited by the author

BASIC CONCEPTS OF A MARKOV-CHAIN

A stochastic process is called a discrete time Markov-chain [6], [7], [8] if it is „memoryless”. It means that the probability distribution of the stochastic process at time n, in other words the current state of the process depends only on the previous distribution, so the distribution at time $(n-1)$ and independent of every prior distribution. This property is usually defined by the following mathematical formula where S_n denotes the state of the process at time n:

$$P\left(S_n \middle| S_{n-1}, S_{n-2}, ..., S_1\right) = P\left(S_n \middle| S_{n-1}\right) \quad (1)$$

The meaning of the expression is that the conditional probability of the event that the process is in state S_n depends only on the previous state S_{n-1} and independent to every previous state. Assuming that this condition is fulfilled, the consecutive probability distributions can be given by the following equations:

$$\boldsymbol{\pi}_{n+1} = \boldsymbol{\pi}_n \mathbf{R}; \; n = 0,1,2.... (2)$$

where row vectors $\boldsymbol{\pi}_0; \boldsymbol{\pi}_1; \boldsymbol{\pi}_2; .....n = 0,1,2....$ are consecutive discrete distributions of the stochastic Markov-process, and matrix **R** is the so-called transition matrix of the Markov-chain. For example let the transition matrix be the following 3×3 matrix

$$\begin{array}{cc} & \textit{to} \\ \textit{from} \quad \mathbf{R} = & \begin{bmatrix} 0,5 & 0,3 & 0,2 \\ 0,5 & 0,1 & 0,4 \\ 0,4 & 0,3 & 0,3 \end{bmatrix} \end{array} \quad (3)$$

This matrix represents a process in which there are three states, and entries of the matrix are transition probabilities from one state to another in the following sense. Rows represent the initial states („from”), columns represent terminal states („to”). Therefor every row is a discrete probability distribution, consequently this matrix is called a row-stochastic Markov-matrix. Assuming that the initial state of the process at time n = 0 is given by the vector $\boldsymbol{\pi}_0 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ than according to formula (2) the Markov-chain can be given by the following sequence of probability distributions:

$$\begin{aligned} \boldsymbol{\pi}_1 = \boldsymbol{\pi}_0 \mathbf{R} &= \begin{bmatrix} 1\,0\,0 \end{bmatrix} \begin{bmatrix} 0,5 & 0,3 & 0,2 \\ 0,5 & 0,1 & 0,4 \\ 0,4 & 0,3 & 0,3 \end{bmatrix} = \begin{bmatrix} 0,5\; 0,3\; 0,2 \end{bmatrix}; \\ \boldsymbol{\pi}_2 = \boldsymbol{\pi}_1 \mathbf{R} &= \begin{bmatrix} 0,5\; 0,3\; 0,2 \end{bmatrix} \begin{bmatrix} 0,5 & 0,3 & 0,2 \\ 0,5 & 0,1 & 0,4 \\ 0,4 & 0,3 & 0,3 \end{bmatrix} = \begin{bmatrix} 0,48\; 0,24\; 0,28 \end{bmatrix}; \qquad (4) \\ \boldsymbol{\pi}_3 = \boldsymbol{\pi}_2 \mathbf{R} &= \begin{bmatrix} 0,48\; 0,24\; 0,28 \end{bmatrix} \begin{bmatrix} 0,5 & 0,3 & 0,2 \\ 0,5 & 0,1 & 0,4 \\ 0,4 & 0,3 & 0,3 \end{bmatrix} = \begin{bmatrix} 0,472\; 0,252\; 0,276 \end{bmatrix}; \; \textit{etc.} \end{aligned}$$

This sequence of probability distributions – stochastic process – describes the behavior of the Markov-chain as time passes.

In the following sections a time series examination and forecasting method will be presented in which the previously illustrated logic of Markov processes will be applied, naturally in a more general context. The basic question in the examined problem is what mathematical concept can be considered as a „state” of a time series, the following question is how can the „transition" be interpreted and finally, what could be a reasonable generalization of the „transition matrix" for a time series. These problems will be studied in the following sections. A Markov-chain is called stationary, if matrix **R** is independent to time, like in this study.

CHARACTERIZATION OF TIME SERIES USING FUZZY LOGIC

The following step is the application of fuzzy logic for time series, in other words the fuzzification process [9], [10]. The basic idea is that not the observed time series data but the yearly change data will be fuzzified. Considering D$Y(n)$ data in the last column of table 1., seven intervals have been chosen for classifying yearly change data. It must be emphasized, that for this construction there are no rigorous rules, these intervals can be constructed arbitrarily, but the length of these intervals must be equal. In the examined case, these intervals and corresponding linguistic variables and the fuzzy sets are as follows:

- [–4000, –3000]: Drastic decrease (fuzzy set A_1)
- [–3000, –2000]: Significant decrease (fuzzy set A_2)
- [–2000, –1000]: Considerable decrease (fuzzy set A_3)
- [–1000, 0]: Slight decrease (fuzzy set A_4)
- [0, 1000]: Slight increase (fuzzy set A_5)
- [1000, 2000]: Considerable increase (fuzzy set A_6)
- [2000, 3000]: Significant increase (fuzzy set A_7)

The following step is choosing suitable membership functions for the accurate definition of the above mentioned fuzzy sets [9], [11]. In this paper the following bell-shaped function is applied.

$$\mu(x) = \frac{1}{1 + c \cdot (x - m)^2}; \quad (5)$$

where m is the center of symmetry – the mean value – and c is a scale parameter. The graphic illustration of this membership function for the same mean value and for various scale parameters can be visualized in figure 2.

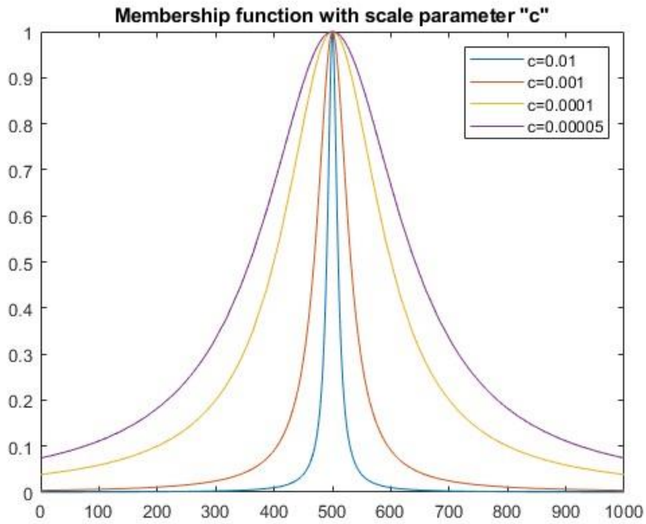

Figure 2. A bell-shaped membership function, given by the formula (5) for m = 500 and for various scale parameters. Source: edited by the author

The advantage of this membership function is that by the scale parameter it can be "tuned" easily. As it can be identified in the graph, the greater the value of the scale parameter, the smaller the outspread of the curve, in other words, the curve is more and more concentrated around the center. This is the most important reason for the application of this mathematical tool, the fuzzy logic, because as it will be clear at the end of this paper, thanks to this tunability the approximation of the time series can be improved. The value of parameter c basically affects the accuracy of the approximation. So this is the reason why fuzzy logic has been chosen for examining a problem like this.

In the further study the above defined membership function will be applied for the seven intervals. The corresponding mathematical formulas are the following:

$$\mu_k(x) = \frac{1}{1 + c \cdot \left(x - m(k)\right)^2}; \; k = 1, 2, \ldots, 7; \qquad (6)$$

where c is obviously the same scale parameter for every fuzzy set, and $m(k)$ is the center of the kth interval. For the corresponding fuzzy sets the notation A_k ($k = 1, 2, \ldots, 7$) will be used. The system of membership functions is displayed in figure 3 with scale parameter $c = 10^{-6}$. This choice of the scale parameter value later will be explained.

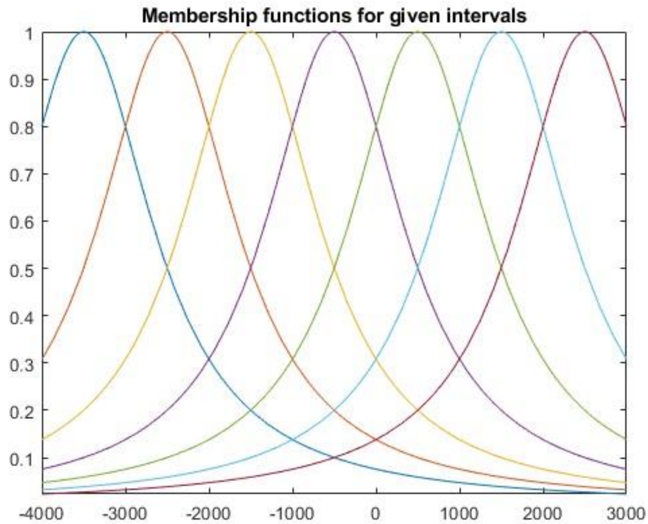

Figure 3. Membership functions for the given seven intervals with scale parameter $c = 10^{-6}$.
Source: edited by the author

The following step is the definition of the "state of the time series". This is a key concept of this paper. The characterization of states of the time series is based on the previously defined fuzzy sets, in other words by the membership functions. More precisely, for every yearly change data, according formula (6) and figure 3. the corresponding membership values must be computed, and a vector of membership values must be determined. The procedure is illustrated in figure 4. for the first three yearly change values (see table 1. column 3).

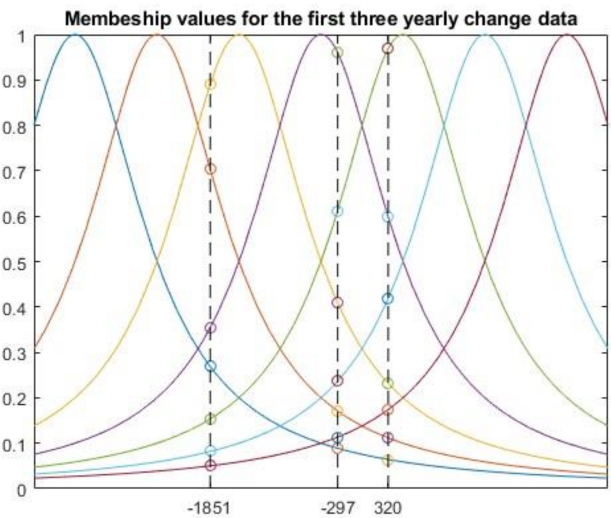

Figure 4. Membership values for the first three yearly change data
Source: edited by the author

For these data the membership values are summarized in the table 2. Obviously the *k*th coordinate of such vector is the membership value that corresponds to fuzzy set A_k.

A_{2001}	0,2689	0,7036	0,8903	0,354	0,1532	0,0818	0,0502
A_{2002}	0,0888	0,1708	0,4086	0,9604	0,6115	0,2365	0,1133
A_{2003}	0,0641	0,1117	0,2319	0,5979	0,9686	0,418	0,1738

Table 2. Membership values for the first three yearly change data
Source: edited by the author

Using the concept of fuzzy logic, these vectors are considered as states of the time series. Furthermore, these states can be characterized by one single fuzzy set, for which the membership value is maximal. Considering table 3. it is clear that in the first three years, the maximal membership values correspond to fuzzy sets A_3, A_4 and A_5 respectively. Following this procedure and concept, in every year the state of the time series can be identified. data are summarized in table 3.

year	Annual number of workplace accidents: $Y(n)$	Yearly change: $DY(n)$	State, described by a fuzzy set
2000	28 220	–	–
2001	26 369	-1 851	A_3
2002	26 072	-297	A_4
2003	26 392	320	A_5
2004	24 355	-2 037	A_2
2005	24 346	-9	A_4
2006	23 038	-1 308	A_3
2007	21 154	-1 884	A_3
2008	22 458	1 304	A_6
2009	18 693	-3 765	A_1
2010	20 123	1 430	A_6
2011	17 448	-2 675	A_2
2012	17 164	-284	A_4
2013	17 361	197	A_5

year	Annual number of workplace accidents: $Y(n)$	Yearly change: $\mathrm{D}Y(n)$	State, described by a fuzzy set
2014	19 787	2 426	A_7
2015	21 165	1 378	A_6
2016	23 027	1 862	A_6
2017	23 387	360	A_5
2018	23 738	351	A_5
2019	24 055	317	A_5
2020	20 366	-3 689	A_1
2021	21 591	1 225	A_6
2022	21 273	-318	A_4

Table 3. States of the time series characterized by fuzzy sets
Source: edited by the author

The following step is considering the time series as a "process", as "transitions between states". Using observations in table 3. and previous classification, it is clear, that the list of transitions in natural order can be given by the formula (7).

$$\begin{aligned} &A_1 \to A_6, \\ &A_2 \to A_4, \\ &A_3 \to A_3, A_4, A_6, \\ &A_4 \to A_3, A_5, \\ &A_5 \to A_1, A_2, A_5, A_7, \\ &A_6 \to A_1, A_2, A_4, A_5, A_6 \\ &A_7 \to A_6, \end{aligned} \tag{7}$$

In this list the observational order doesn't matter, because, as it will be clear in the next section, the fuzzy arithmetic operations, that will be applied, are commutative, therefore only every observed transition must be taken into account, but the order is indifferent. Furthermore due to the same reason, thanks to the nature of the applied mathematical operations, that are max and min operations, if transition $A_k \to A_j$ can be observed more than once, in the list of transitions it is enough considering it only once.

APPLICATION OF THE CONCEPT OF MARKOV CHAIN

In this section on the one hand the proposed application of the concept of Markov-chain – formula (2) –, and on the other hand the proposed integration of two great theories, Markov-process and fuzzy logic, will be presented [4], [5].

The basic problem is the construction procedure of the transition matrix **R**. In this application, the meaning of entries of the transition matrix are not probabilities like in the classic Markov-process, but in some sense, fuzzy membership values. The basic goal is characterizing the whole time series by one matrix, this will be matrix **R**, in which every mathematical property, every necessary attribution will be summarized for every observed transition. The construction procedure of matrix **R** is as follows.

Like in table 2. for every fuzzy set A_k ($k = 1, 2, \ldots, 7$) a seven-component vector will be assigned, which components are membership values that correspond to the mean value m_k of the kth interval. Table 4. contains data, that are obtained using formula (6).

A_1	1	0,5	0,2	0,1	0,059	0,036	0,027
A_2	0,5	1	0,5	0,2	0,1	0,059	0,036
A_3	0,2	0,5	1	0,5	0,2	0,1	0,059
A_4	0,1	0,2	0,5	1	0,5	0,2	0,1
A_5	0,059	0,1	0,2	0,5	1	0,5	0,2
A_6	0,036	0,059	0,1	0,2	0,5	1	0,5
A_7	0,027	0,036	0,059	0,1	0,2	0,5	1

Table 4. Characterization of fuzzy sets by membership values, that correspond to mean values of intervals. Source: edited by the author

Using this table and transitions, that are highlighted by formula (7), for every transition

$A_k \rightarrow A_j$ a "one-step" transition matrix is generated, denoted by $\mathbf{R}_{kj}$, which matrix contains every mathematical information about the transition. This matrix is constructed by the fuzzy-arithmetic generalization of the dyadic product [9], [10], [11] of vectors that correspond to A_k and A_j, in this order:

$$A_k \rightarrow A_j \Rightarrow \mathbf{R}_{kj} = A_k^T \cap A_j \in \mathbf{R}^{n\times n};$$
$$\text{that is} \qquad (8)$$
$$\mathbf{R}_{kj}(m,p) = \min\left(A_k(m); A_j(p)\right)$$

Where m and p denotes the row and column index of the entry respectively. The reason for this definition is that in fuzzy arithmetic, operation product is substituted by min operator. Superscript "T" denotes the transpose of the vector. Simply to say, a column vector must be multiplied by a row vector. This operation yields a 7×7 matrix for one specific transition.

Finally, considering every transition, given by (7), using formula (8) every “one-step transition matrix” must be generated, and by definition, transition matrix **R** of the whole time series will be the sum of every such “one-step matrix”. Considering the fact, that in fuzzy-arithmetic, addition is substituted by max operation, the mathematical formula for the construction is as follows:

$$\mathbf{R} = \bigcup_{(k,j)} \mathbf{R}_{kj} \in \square^{n\times n} \quad \text{that is} \quad \mathbf{R}(m,p) = \max_{(k,j)}\left(\mathbf{R}_{kj}(m,p)\right) \qquad (9)$$

Performing every operation, entries of the obtained transition matrix is summarized in the following table:

0,2	0,2	0,5	0,5	0,5	1	0,5
0,2	0,5	0,5	1	0,5	0,5	0,5
0,2	0,5	1	1	0,5	1	0,5
0,5	0,5	1	0,5	1	0,5	0,5
1	1	0,5	0,5	1	0,5	1
1	1	0,5	1	1	1	0,5
0,5	0,5	0,5	0,5	0,5	1	0,5

Table 5. Entries of the transition matrix **R**
Source: edited by the author

Using this transition matrix, to the analogy of classic Markov-process, the forecasting procedure can be given by the following mathematical formula:

$$\begin{aligned} &A_n = A_{n-1} \circ \mathbf{R}; \\ &A_{n-1}: \text{observation in year } (n-1) \qquad (10) \\ &A_n: \text{forecast for year } n \end{aligned}$$

where matrix **R** is given by table 5. and operation “o” denotes the fuzzy-type matrix multiplication, using min-max operations instead of multiplication and addition [9], [10], [12].

Considering formula (10) it is obvious, that the operation yields a row vector as a prediction, which row vector must be considered as a fuzzy set as well. Summarizing the essence of the article, according to the Markovian approach, every state of the time series is given by a fuzzy set, the prediction is also defined by a fuzzy set, the process predicts the yearly change of the time series, and the transition matrix is constructed using membership values and fuzzy operations.

The last step is the defuzzification procedure of obtained predictions. For defuzzification the proposed method is one of the most frequently applied technique, which is the centroid method. This can be formulated as follows.

$$\text{forecast for year } (n) = \frac{\sum_{k=1}^{7} \mu_k \left(A_n(k) \right) m(k)}{\sum_{k=1}^{7} \mu_k \left(A_n(k) \right)};$$

$$m(k): \text{the mean of } k\text{th interval} \tag{11}$$

$A_n(k)$ = the kth component of the predicted fuzzy set for year n

The reason for choosing exactly the centroid method is clear. The expression, given by (11) can also be given in the following form

$$\text{forecats}(n) = \frac{\mu_1\left(A_n(1)\right)}{\sum_{k=1}^{7}\mu_k\left(A_n(k)\right)} m(1) + \frac{\mu_2\left(A_n(2)\right)}{\sum_{k=1}^{7}\mu_k\left(A_n(k)\right)} m(2) + \ldots + \frac{\mu_7\left(A_n(7)\right)}{\sum_{k=1}^{7}\mu_k\left(A_n(k)\right)} m(7) \tag{12}$$

in which a basic quantity in probability theory can be identified, which is the expected value [6], [7], since in the sum (11) fractions form a discrete probability distribution. This approach shows a more stronger relation to the theory of Markov-processes. The outcome of the defuzzification process, the predicted value will be the forecast for the following yearly change in the time series, and not directly for the following value in the series.

TIME SERIES FORECASTING FOR THE EXAMINED DATA SERIES

In this section, the application of the proposed method will be presented for the specific time series, that is given by the table 1.

Applying the Markovian iteration given by (10) using transition matrix **R** summarized in table 5., for states that are characterized by fuzzy sets illustrated in table 2. and finally using scale parameter $c = 10^{-6}$, the original time series, and predictions, including the following three years, are depicted in figure 5.

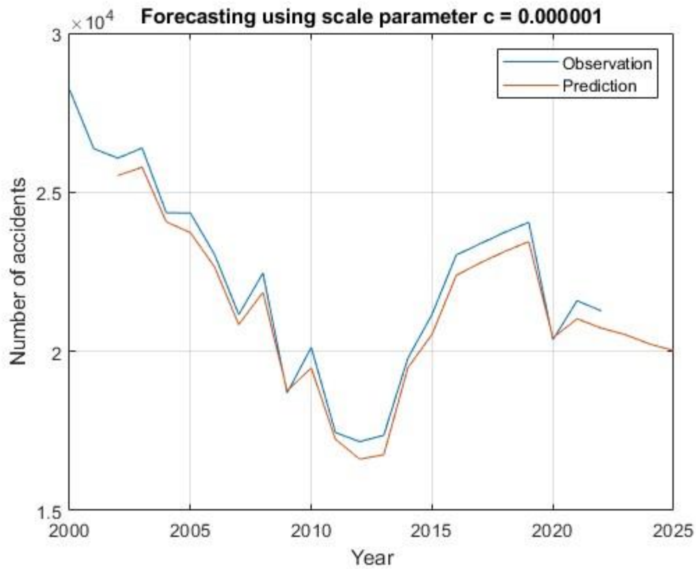

Figure 5. Forecasting the time series using fuzzy-Markov process
Source: edited by the author

Predictions can also be seen in the table, not only for the following three years, but for every year after 2001, because comparing observations and predictions is a fundamental test for efficiency of the prediction method. Data are summarized in table 6.

Year	Observation	Prediction	Relative error (%)
2000	28 220	–	–
2001	26 369	–	–
2002	26 072	25525	2,098036
2003	26 392	25788	2,288572
2004	24 355	24067	1,182509
2005	24 346	23728	2,538405
2006	23 038	22652	1,675493
2007	21 154	20843	1,470171
2008	22 458	21856	2,680559
2009	18 693	18761	0,363773
2010	20 123	19476	3,215226

Year	Observation	Prediction	Relative error (%)
2011	17 448	17237	1,209308
2012	17 164	16614	3,204381
2013	17 361	16751	3,513622
2014	19 787	19488	1,511093
2015	21 165	20535	2,976612
2016	23 027	22385	2,788031
2017	23 387	22786	2,569804
2018	23 738	23136	2,536018
2019	24 055	23451	2,510912
2020	20 366	20425	0,289699
2021	21 591	21023	2,630726
2022	21 273	20731	2,547831
2023	–	20523	–
2024	–	20231	–
2025	–	20023	–

Table 6. Comparison of observations and predictions, and predictions for the following three years
Source: edited by the author

In the table the relative error is also highlighted in every forecasted year, according to the formula

$$\text{Relative error (\%)} = \frac{|\text{Observation} - \text{Prediction}|}{\text{Observation}} \cdot 100 \qquad (13)$$

Considering the last column, it can be seen, that the relative error of the forecast for existing observations, which is a measure for accuracy, is mostly between 2 and 3 percentage, which accuracy is satisfactory. According to these predictions, the number of workplace accidents will decrease in the following three years.

The remaining question is examining the effect of the scale parameter. As it was mentioned earlier, this is a key point in the application of the proposed algorithm, this is one of the reasons for choosing fuzzy logic. In figure 6. the system of membership functions and the corresponding forecast can be visualized for various scale parameters.

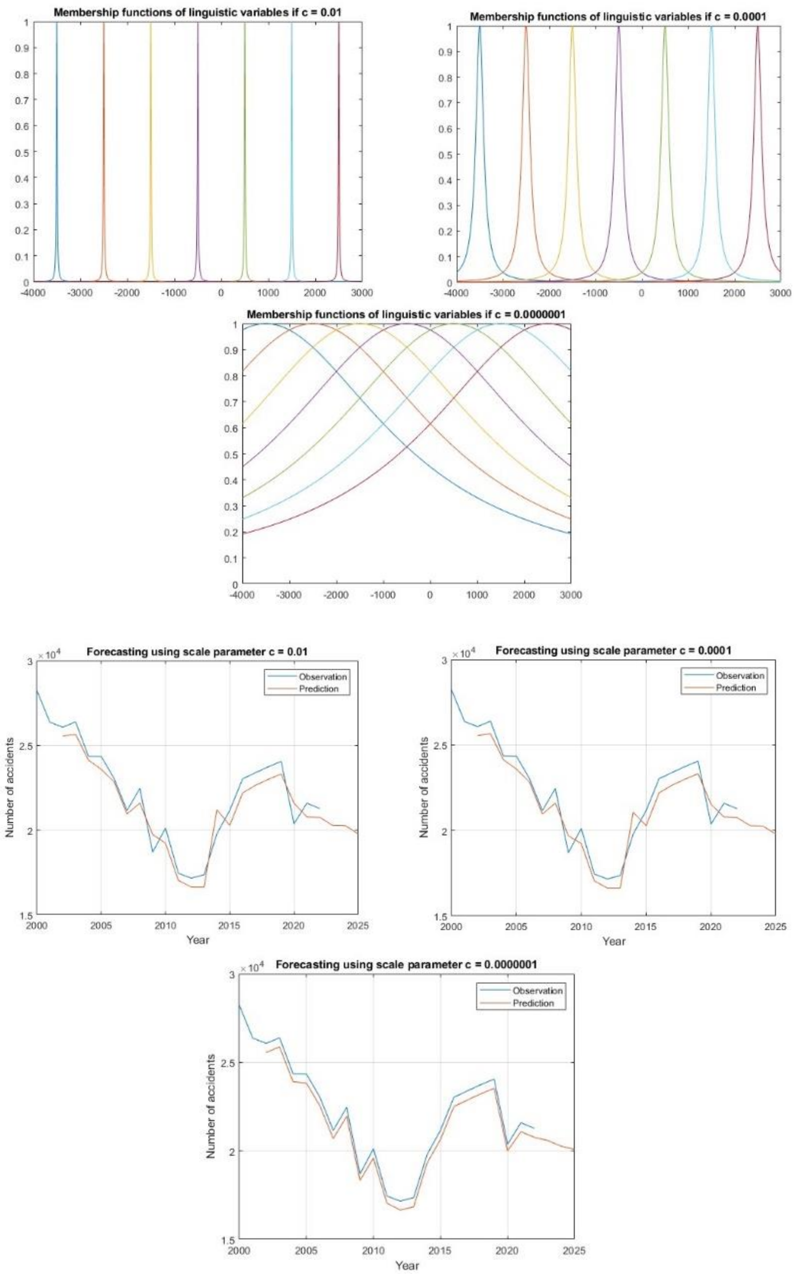

Figure 6. The system of membership functions and corresponding predictions for various scale parameters: from left to right c = 0,01; 0.0001; 0.0000001
Source: edited by the author

As it can be seen at first sight, that for extremely narrow (c = 0.01) and for extremely wide (c = 0.0000001) membership function, the accuracy of the forecast is much worse, and it is also valid for the medium scale parameter c = 0.0001. Computing relative error data for these predictions, the obtained results also prove, that these predictions are worse than the forecast obtained for the scale parameter $c = 10^{-6}$ and highlighted in table 6. Considering satisfactory accuracy of the prediction for this parameter, the forecast for the following three years can be accepted. Therefore it can be stated, that among the examined four scale parameters, the best result, the best accuracy is obtained for $c = 10^{-6}$. If someone would like to find the optimal choice in classic sense, for example one global optimization procedure, like genetic algorithm could be applied. But in this study, considering the high accuracy, it is not necessary.

CONCLUSIONS

In this paper an efficient time series forecasting method was presented and has been applied for a real time series, which can be considered as a security problem. The basis of the proposed tool is the unification of two theories, Markov-chains and fuzzy logic. Results that are presented in figure 5. and table 6. are convincing. The approximation of observed data by the forecasting procedure turned out to be extremely accurate, since relative error of the approximation is around 2-3%. Therefore the predictions for the near future can be accepted.

The essence of this paper is that tools and concepts of fuzzy logic can be applied for this situation as well, this theory also works under such circumstances, and happened to be very efficient. The point is that not observed data in the series must be fuzzified but the change of observed data, therefore predictions also can be given for the change of data. Using these concepts a time series can be characterized by states, and states are given by fuzzy sets. Considering the role of scale parameter, the effect of fuzzy theory is clear. For the predicting procedure, for the transition process the other tool, the concept of the theory of Markov-chain is applied, which also produces fuzzy sets as predictions. Every fuzzy set can be transformed to “crisp” value using any defuzzification process.

Considering further study, one can analyze other type of membership functions that are not bell-shaped, for example triangular or trapezoidal functions. If higher accuracy is a requirement, some global optimization method can be applied for finding “the best scale parameter”. Furthermore, another challenge could be a non stationary Markovian model, in which the transition matrix **R** depends on time.

REFERENCES

[1] Box, Jenkins, ‘Time Series Analysis, Forecasting and control’, Wiley, 2016.
[2] Lynwood A., Johnson, L.A, Montgomery, D.C, and Gardiner, J.S., Forecasting and Time Series Analysis. 2nd Edition. McGraw-Hill,Inc, 1990.
[3] Box, Jenkins, Reinsel, Ljung: ‘Time Series Analysis’, Springer, 2016.
[4] Guangming: Fuzzy Markov chains based on the fuzzy transition probability, IEEE Chinese Control and Decision Conference, 2014
[5] Avrachenkov, Sanchez: Fuzzy Markov chains, Fuzzy Optimization and Decision Making, 2002
[6] Ross: Stochastic processes, John Wiley and sons, USA, 1996
[7] Karlin - Taylor: An introduction to stochastic modeling, Academic Press, London, 1998
[8] Knill: Probability and Stochastic Processes with Applications, Overseas Press, India, 2006

[9] László, Pokorádi ; Sinan, Kocak ; Edit, Tóth-Laufer: Fuzzy Failure Modes and Effects Analysis Using Summative Defuzzification Methods, ACTA POLYTECHNICA HUNGARICA 18 : 9 pp. 111-126. , 16 p. (2021)
[10] Kóczy, Tikk: Fuzzy rendszerek, Typotex, Budapest, 2001
[11] Paasino-Yurkovich: Fuzzy control, Addison-Wesley, USA, 1998
[12] Meghdadi-Akbarzadeh: Probabilistic Fuzzy Logic and Probabilistic Fuzzy Systems, IEEE International Fuzzy Systems Conference, 2001